INNOVATIVE MODULAR BUILDING MANUFACTURED USING STRUCTURAL BLACK POPLAR PLYWOOD PANELS

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This article presents a small prefabricated building with constructive systems not usually utilized in Spain, together with the structural checking needed before its construction. The principal material is the black poplar plywood panel. These wood-derived panels may enhance building sustainability and lighten their weight, showing new business opportunities because of the possibility of mass production. This also allows companies to respond to large orders in shorter periods of time, such as critical situations after some recent catastrophes. The construction has to table with the worst environmental conditions in Spain, which are defined by the Spanish Building Technical Code (CTE). For that reason, the national standard mentioned and other recommendations from the American Engineered Wood Association were used in design and structural checking processes. Another important requirement is that the construction has also to be easy to set up by non-qualified workers.

Its constructive solution consisted of a bearing wall that supports a roof with two slopes. A wide inner bearing wall supports the weight of the tile roof.

Maximum moments and displacements generated were obtained using the Navier and Levy models based on plate theory. These calculations allowed us to demonstrate the significant resistance of the construction proposed.

Keywords: *Laminated composite materials; Plywood panel; Modular building; Spanish Building Technical Code; Plate theory*

EDIFICACIÓN MODULAR INNOVADORA CONSTRUIDA MEDIANTE EL USO DE TABLERO ESTRUCTURAL DE CONTRACHAPADO DE MADERA DE CHOPO

Este artículo presenta un caso de pequeña edificación prefabricada con métodos constructivos no habituales en España y las comprobaciones estructurales a realizar para su ejecución. Para su construcción se utiliza tablero estructural de contrachapado de chopo. Este material permite mejorar la sostenibilidad de los edificios, aligerar su peso y abre un nuevo horizonte de oportunidades ante la posibilidad de ser producido en masa. Esto permitiría responder a pedidos de gran volumen en cortos periodos de tiempo, como en reconstrucciones tras ciertas catástrofes. Esta construcción debe poder colocarse en cualquier lugar de España. Por ello se han considerado las condiciones más desfavorables contempladas en el Código Técnico de la Edificación (CTE). Las comprobaciones estructurales se realizan según el CTE y la Asociación Americana de Ingeniería de la Madera. También se requiere que sea de fácil montaje para realizarlo sin personal especializado.

La solución constructiva adoptada consiste en un muro portante que actúa de cerramiento y una cubierta a dos aguas, apoyada sobre un tabique interior.

Para la obtención de los esfuerzos y deformaciones máximas se ha utilizado la teoría de placas mediante los métodos de Navier y de Levy. Estos cálculos permiten destacar la resistencia constructiva de la edificación.

Palabras clave: *Materiales laminados; Contrachapado de madera; Edificación modular; Código Técnico de la Edificación; Teoría de placas*

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1. Introduction

Nowadays, plate's theory is one of the most important theories in Structural Engineering. This is mainly due to the high number of elements defining the behaviour of plates. These are flat structural elements, which may be approximated by a two-dimensional surface and normally work with bending forces.

Using terms of elasticity, a plate is known as a sheet. This is a cylindrical body in which its height is much shorter than the characteristic dimensions of its base. A sheet is considered thin whether the relation between thickness and one characteristic dimension (x or y) is much smaller compared with unity. In contrast to a plane strain state, loads are essentially normal to the median plane. Median plane is called the common place of points equidistant from both surfaces. At any point in the median plane, a perpendicular line can be drawn that defines the direction of the normal vector to the surface at that point. In this article, the median plane is assumed as a continuous surface, which has only a smooth variation.

The basic hypothesis assumptions taken into account are the followings:

- The movements of the median plane of a surface plate are smaller than the thickness of the plate, so the vertical displacements of the points of the median plane do not depend of z direction.
- Any straight line that initially is normal to the median surface will continue being straight and normal to that surface after the flexural stress. That means that points of the median plane only suffer vertical displacement.
- Normal stress is much smaller compared with other stress and it can be neglected.

Spain and France are two important countries in manufacture of black-poplar wood (Cooper 2002). The most well known product derived from black-poplar wood harvested from both countries is the plywood panel. However, up to now, plywood panels have mostly been used as decorative purposes as boards for construction processes. Thus the use of plywood panel as load-bearing wall for small constructions is a promising market for companies in the European Union (EU) (Navarrete-Castillo 2008).

As in Navarrete-Castillo (2008) it can be seen, plywood panels offer many possibilities for European companies. In specially Spanish and French enterprises can exploit this market due to they are two of the most important countries in black-poplar wood manufactures, Cooper (2002).

Previous work Alía et al. (2012) already showed the methods that can be used for obtaining plywood panel properties. The aim of this study is to demonstrate how by combining with other materials these features can be improved to implement sandwich plywood panels as structural element. In order to show it a practical case is developed. This practical case contains the structural checks that have to be performed on a small construction using this material. The construction is carried out using the structural panel as load-bearing Wall.

Loads that have to be tested are calculated using the CTE. It is remarkable that the edification can be placed anywhere in Spain. These charges are combined to get the most unfavourable assumptions. Two different checks must be done. The first one checks the last resistance of the board. The second one focuses on the visual aspect of the building, setting maximum displacements for each one of the resistive elements. In this article floor, wall and roof will be checked separately.

2. Methodology

2.1. Materials

Hybrid materials combine two or more compounds in order to develop new characteristics that each one of the materials separately couldn't offer. Materials that are made of fibbers are one of the kinds of hybrid materials, but there are many others as sandwich materials in witch this study focuses. The combination of different raw materials and the different fabrication methods allow getting new materials with many diverse typologies.

A structural plywood panel is a sandwich material that combines two materials in a defined geometry. This geometry is configured using two plywood faces and a polystyrene core. Using this configuration the flexural rigidity increases but the weight is reduced. The introduction of the core between the faces increases the inertia moment of the section producing a structure able to resist flexion and buckling efficiently.

Not only mechanical properties are improved as mentioned previously. Using these hybrid materials a high performance in terms of thermal and acoustic isolation can be obtained.

2.1.1. Elasticity modulus

In order to obtain the mechanical properties of the sandwich panel, Ashby (2005) is used. The book gives an equation that allows evaluating the resistant capacity of both materials in consonance.

$$\frac{1}{E_{f,0}} = \frac{1}{E_f\left[(1-(1-f)^3) + \frac{E_c}{E_f}(1-f)^3\right]} + \frac{B_1}{B_2}\left(\frac{d}{L}\right)^2 \frac{(1-f)}{G_c} \qquad (1)$$

Where T is the thickness of the faces, d is the normal thickness, c is the thickness of the soul, L is the length of the panel, E_f is the elasticity modulus of the faces, E_c is the elasticity modulus of the core, G_c is the shear modulus of the core, f can be obtained using $f = \frac{2 \cdot t}{d}$ and B_1 and B_2 are coefficients that depend of the boundary conditions. In case of a fixed ended panel, $B_1 = 384$ and $B_2 = 8$. If all sides of the panel are simply supported, $B_1 = 384/5$ and $B_2 = 8$.

2.1.2. Flexural Strength

As happened with elasticity modulus Ashby (2005) is used with the purpose of determine the strength of the sandwich panel.

$$\sigma_{flex} = (1-(1-f)^2)\sigma_f + (1-f)^2\sigma_c \qquad (2)$$

Where σ_f is the strength of the faces and σ_c is the strength of the core.

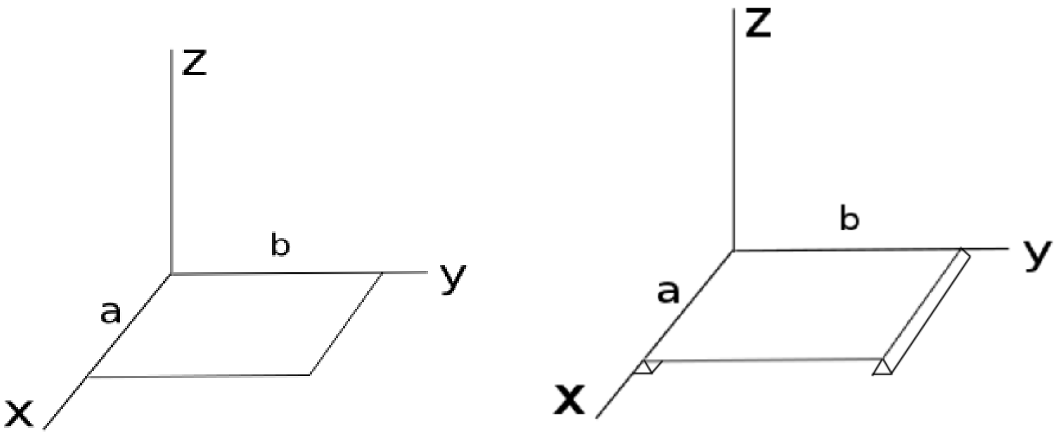
2.1.3. Compressive Strength

As in previous paragraph the equation for hybrid materials of Ashby (2005) is used. Taking into account the volumetric friction of each one of the materials.

$$\sigma_{comp} = f_f \cdot \sigma_f + f_c \cdot \sigma_c \qquad (3)$$

2.2. Plates theory

Figure 1. Navier Solution (left) and Levy Solution (right)



2.2.1. Navier

As shown in Blázquez (2004), Navier method uses a double series equation of the movements of the plate witch satisfies boundary conditions. By substitution into the differential equation of the plates, conditions that have to be met by the series coefficients are obtained. In this section only rectangular plates under simply supported boundary conditions will be considered. The solution originally developed by Navier in 1820 will be studied.

A four sides simply supported plate is supposed. These sides are defined by $x = 0$, $x = a$, $y = 0$ and $y = b$. Over the plate a transversal charge $q_z(x, y)$ acts. This charge will be called $p(x, y)$. The equation that must be solve is:

$$\nabla_w{}^4 = \frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \cdot \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D} \qquad (4)$$

Three boundary conditions must be satisfied. First of them is $w = 0$. The second one in $x = 0$ and $x = a$ is $\frac{\partial^2 w}{\partial x^2} = 0$. The last one is $\frac{\partial^2 w}{\partial y^2} = 0$ in $y = 0$ and $y = a$.

The real plate is supposed as part of another fictitious plate four times greater and it is submitted to an odd charge. This means; $p(x, y) = -p(-x, y) = -p(x, -y) = p(-x, -y)$. Using this conditiones movements can be also considered as an odd function, $w(x, y) = -w(-x, y) = -w(x, -y) = w(-x, -y)$. The period in the x direction is $T = 2 \cdot a$, and $T' = 2 \cdot b$ in the y direction. Using $w = \frac{\pi}{a}$, $w' = \frac{\pi}{b}$, in order to solve the problem using Fourier series, it is considered that the charge can be decomposed as:

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} c_{mn} \cdot \sin \frac{m \cdot \pi \cdot x}{a} \cdot \sin \frac{n \cdot \pi \cdot y}{b} \qquad (5)$$

2.2.2. Levy

This method described in Blázquez (2004), uses a two-sides simply supported plate $y = 0$ and $y = b$. Leaving boundary conditions of the two other sides undefined for the moment, charge is defined by:

$$p(x, y) = g(x) \cdot h(y) \quad (6)$$

Function $h(y)$ is developed by a Fourier simply series to odd functions. Admitting that the plate can be extended in y negative direction in order to have a double plate. A $p(x, -y) = -p(x, y)$ charge It is applied in the fictitious part.

$$h(y) = \sum_{m=1}^{\infty} b_m \cdot \sin \frac{m \cdot \pi \cdot y}{b} \quad (7)$$

2.3. Structural checks

All the elements must be checked against two different situations. First one checks the ultimate strain resistance of the material. The second one tests that the maximum displacements generated are less than maximum permissible.

2.3.1. Ultimate strain check

Bearing wall

For this element two different checks are carried out. First of them is made using the CTE, but without including buckling effect. The second one using the criteria established by "The Engineered Wood Association" that includes the buckling effect. Maximum moments are obtained using plates theory. After that the stresses are calculated using:

$$\sigma_c = \frac{N}{A} \quad (8)$$

Where N is the compressive charge and A is the resistive area.

$$\sigma_f = m \cdot \frac{6}{t^2} \quad (9)$$

Where m is the maximum moment and t the thickness of the plate

- CTE

The equations supplied by the CTE are:

$$\left(\frac{\sigma_c}{\sigma_{comp}}\right)^2 + \frac{\sigma_{f,0}}{\sigma_{flex,0}} + k_m \cdot \frac{\sigma_{f,0}}{\sigma_{flex,0}} \leq 1 \quad (10)$$

$$\left(\frac{\sigma_c}{\sigma_{comp}}\right)^2 + k_m \cdot \frac{\sigma_{f,0}}{\sigma_{flex,0}} + \frac{\sigma_{f,0}}{\sigma_{flex,0}} \leq 1 \quad (11)$$

Where σ_c is the maximum compressive stress generated, $\sigma_{f,0}$ is the maximum parallel flexural stress generated, $\sigma_{f,90}$ is the maximum normal flexural stress generated and k_m is a coefficient that takes into account the stress redistribution. For this case is 1.

- Engineered Wood Association

First of all the maximum admissible charge must be obtained using the following equation:

$$P_{max} = \frac{3.619 \cdot E \cdot I}{144 \cdot L^2} \qquad (12)$$

Where I is the inertia moment, L is the length of the panel and E is the elasticity modulus.

After of that this equation must be satisfied:

$$\frac{N}{P_{max}} + \frac{\sigma_{f,0}}{\sigma_{flex,0}} + k_m \cdot \frac{\sigma_{f,0}}{\sigma_{flex,0}} \leq 1 \qquad (13)$$

Ceiling and floor

These plates should be checked against flexural stresses. As in previous section moments and stresses are obtained. All sides of roof panels are simply supported. This checked is carried out using the following equations provided by the CTE.

$$\frac{\sigma_{f,0}}{\sigma_{flex,0}} + k_m \cdot \frac{\sigma_{f,0}}{\sigma_{flex,0}} \leq 1 \qquad (14)$$

$$k_m \cdot \frac{\sigma_{f,0}}{\sigma_{flex,0}} + \frac{\sigma_{f,0}}{\sigma_{flex,0}} \leq 1 \qquad (15)$$

2.3.2. Maximum displacement check

For this check, maximum permissible displacement must be obtained. In this study the criteria fixed is:

$$\delta_{max} = \frac{L}{300} = \frac{250}{300} = 0.83cm \qquad (16)$$

After that plates theory is applied for each one of the situations in order to obtain the maximum displacements generated.

3. Results

3.1. Data of raw materials

Properties of the source materials are describes in the tables below.

Table 1. Plywood Panel Properties

Property	Value (N/mm^2)
Parallel elasticity modulus: E_0	4320
Normal elasticity modulus: E_{90}	5542
Parallel strength: σ_0	19.14
Normal strength: σ_{90}	25.88

Table 2. Polystyrene Properties

Property	Value (N/mm^2)
Elasticity modulus: E_p	12
Shear modulus: G_p	8
Strength: σ_p	0.5

3.2. Sandwich Panel properties

Using the methodology describes above the properties obtained for the sandwich panels are shown:

Table 3. Sandwich Panel Properties

Property	Simply supported sides (N/mm^2)	Fixed ended (N/mm^2)
Parallel elasticity modulus: E_0	1650	526
Normal elasticity modulus: E_{90}	613	613
Parallel flexural strength: $\sigma_{flex,0}$	13.29	13.29
Normal flexural strength: $\sigma_{flex,90}$	17.92	17.92
Compressive strength: σ_{comp}	7.3	7.3

3.3. Case of study

Table below shows the results obtained applying structural checks.

It can be seen that values shown in results column do not reach the unit; so all justifications meet safety standards. It is remarkable that when flexural and compressive stresses are applied simultaneously is the critical situation.

It is also shown in the table that maximum displacements generated are smaller than the maximum allowed value. However they are closer to the limit, specially the floor case more than the rest of them.

Table 4. Structural Results

	N (kN)	Pmax (kN)	σ_c $(\frac{N}{mm^2})$	$\sigma_{f,0}$ $(\frac{N}{mm^2})$	$\sigma_{f,90}$ $(\frac{N}{mm^2})$	Charge max. displacement check	Result Ultimate strain check	Maximum displacement (cm)
Bearing wall (CTE)	-	-	3.48	0.6	0.85	3.97	0.32	0.6
Bearing wall (CTE)	0.29	3.97	-	0.6	0.85	-	0.17	-
Ceiling	-	-	-	0.6	0.19	2.17	0.11	0.19
Floor		--	-	1.25	1.77	3.52	0.19	0.81

4. Conclusions

The case described above demonstrates the capacity of plywood panel as structural element for small constructions. The experiments have satisfied both by the Spanish CTE and the American Wood Association criteria, showing this composite opens up a wide range of possibilities for the development of new construction methods.

We conclude that plywood plates posses much less resistance in combined flexural and compressive stresses applied to their non-resistant face. This is due to the fact that wood is an orthotropic material. Thus, it is crucial the correct adjustment and use of the panels at buildings. However, whether the distribution of fibbers should be considered to obtain a symmetrical board because of the significant increase in its flexural capacity.

This study only takes into account the combination of poplar plywood with polystyrene. The material of the core is chosen due to its isolation properties, but sandwich panels can be modify to other configurations in order to get different properties.

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