APPLICATION OF RESPONSE SURFACE METHODOLOGY FOR THE OPTIMIZATION OF WELDING PARAMETERS AS A FUNCTION OF THE RESULTING THERMAL DISTRIBUTION.

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Abstract

One of the largest difficulties traditionally found in stainless steel constructions has been the execution of welding parts in them. Nowadays, the technology available permits the use of arc welding processes for that application with certain advantage. Response surface methodology is used to optimize a process in which the variables that take part in it are not related to each other by a mathematical law. Therefore, an empiric model must be formulated. With this methodology the optimization of one selected variable may be done. In the case of welding, it allows to determine the best operation conditions and to minimize the probability of the joint having defects. In this research, an optimization of TIG welding parameters is made as a function of temperature distribution in to the joint: current, voltage, arc efficiency and welding speed.

***Keywords**: Response surface methodology, welding, thermal modelling, statistical design, ANOVA*

1. Introduction

The response surface methodology (RSM) is a collection of techniques that allow the researcher to inspect a response that can be displayed as a surface, when the experiments investigate the effect of varying quantitative factors in the values that a dependent variable or response takes (Box et., 1951).The RSM is a set of mathematical and statistical techniques that are useful for modeling and analysis in applications where a response of interest is influenced by different variables and the objective is to optimize this response (Garcia-Diaz, 1995).

One of the difficulties encountered in building stainless steel structures has been the joints of structural profiles. At present, welding joints is applicable due to the advantages inherent in this procedure. However, the temperatures reached are high because the process involves joining with a fusion of material, which can modify the mechanical, chemical and aesthetic properties of the material. In the process of welding with high heat flow, in addition to the fusion zone, the heat affects the areas adjacent to it, HAZ, the size of which depends on the extent of micro-structural transformations that take place.

The concept of cooling time between 800 º C and 500 ° C, symbolized as "$t_{8/5}$" is widely accepted. The factor T $t_{8/5}$ is a parameter that can be controlled more easily in automatic processes than in manual ones. The principle is based on the fact that for certain conditions of welding, the thermal cycle at points of the heat-affected zone, HAZ, will have different maximum temperature values, according to the distance the points are located over the base metal with respect to the centre of the weld cord, but the cooling time between 800 º C and 500 ° C is approximately equal if the welding parameters remain constant.

To optimize the cooling time, different analytical methods can be used and, in particular, Response Surface Methodology (RSM), which allows optimizing a response function subject to different independent variables. It is an objective of any experimental design applied to the optimization of a process, to be able to study the influence of different operating variables, both in regard to the variability of the responses as well as to its central tendency, always carrying out the minimum number of possible experiments. The idea is to establish the theoretical mathematical model that relates them by carrying out the minimum number possible of experiments (Montgomery 1995).

To determine the temperature field in a welded joint, analytical and numerical methods for solving differential equations can be employed which define the thermal diffusion process. The temperature data considered in the current work were obtained using a finite difference method based on the physical definition of the two-dimensional heat flow, considering the changes of state and determining the liquid fraction at each point of the cord (Estrems et al., 2007).In this work values of the temperature field have been obtained at points located at a distance of 1.5 mm from the weld cord axis.

2. Experimental procedure and results

The response surface methodology is applied in conjunction with the factorial experimental design. The approach consists in using the design of experiments to determine which variables are influencing the response of interest. Once these variables have been identified, a rough estimate of the response surface is obtained by means of factorial models. This response surface is used as a guide to gradually vary the controllable factors affecting the response in order to improve said value. One of the simplest forms of response surface is given by first order linearity, where "ε" is the experimental error, as indicated in equations (1), (2) and (3).(Box et al., 1951).

$$\eta = f(x_1, x_2) + \varepsilon(1)$$

$$y = \beta_o + \sum \beta_i x_i + \varepsilon \qquad (2)$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon(3)$$

Where x_1, x_2, ..., x_k are the independent variables and β_0, β_1, ..., β_k are the regression parameters of the surface estimated from the experimental data.

The factors considered as variables in the welding process, the electric arc power *P,* the welding speed v and the performance of the welding process *η.* The response of the method is affected by changing the levels of the factors. The response is $t_{8/5}$, and the interest of the method is to optimize that value.

The response function can be represented by a polynomial equation, which in the case studied fits a first-degree polynomial. In accordance with the RSM, the model obtained is that which is shown in the equation (4).

$$\mathbf{t_{8/5} = \beta_o + \beta_1 P + \beta_2 v + \beta_3 \eta + \varepsilon} \qquad (4)$$

The relationship between the factors and levels considered is indicated in Table 1

Factors	Levels		
	-1	0	1
Power	910.00 W	1720.00 W	2530.00 W
Speed	3.30E-03 m/s	3.73E-03 m/s	4.16E-03 m/s
Efficiency	0.4	0.6	0.8

Table 1. Levels of the variables of the arc welding process

With the combination of the values of the variables listed in Table 1 the matrix of experiments has been made, taking into account the codified values of the variables according to the levels (-1, 0, 1), these levels correspond to the minimum, mean and maximum values of *P, v* and *η,* these values have been calculated using the experimental procedure (Miguel et al., 2008), therefore, the input values for the thermal simulation are actual values used in welding procedures. Table 2 shows the experiments selected in order to obtain the matrix of experiments.

Factors	Power	Speed	Efficiency
Experiment	**A**	**B**	**C**
1	0	-1	1
2	0	0	0
3	1	0	1
4	0	-1	-1
5	-1	-1	0
6	-1	0	1
7	1	1	0
8	1	0	-1
9	-1	1	0
10	0	0	0
11	1	-1	0
12	0	1	-1
13	0	1	1
14	0	0	0
15	-1	0	-1

Table2. Codification of the factors –levels

Table 3 shows the values for intensity, welding stress, welding speed and thermal efficiency of the process for each of the experiments, thus the matrix of experiments is obtained, showing the values of the factors for each of the experiments conducted.

Experiment	Power (I*V)	Speed (m/ sec)	Efficiency
1	1720.00	0.0020833	0.8
2	1720.00	0.00312465	0.6
3	2530.00	0.00312465	0.8
4	1720.00	0.0020833	0.4
5	910.00	0.0020833	0.6
6	910	0.00312465	0.8
7	2530	0.004166	0.6
8	2530	0.00312465	0.4
9	910	0.004166	0.6
10	1720	0.00312465	0.6
11	2530	0.0020833	0.6
12	1720	0.004166	0.4
13	1720	0.004166	0.8
14	1720	0.00312465	0.6
15	910	0.00312465	0.4

Table 3. Values of the matrix of experiments

The RSM solves the matrix equation relating the $t_{8/5,}$ depending on the welding parameters used to perform the experiments, selected from the matrix of experiments indicated in equation (5)

$$[\mathbf{t_{8/5}}] = [\mathbf{X}][\boldsymbol{\beta}] \qquad (5)$$

Where the matrix [X] represents the independent terms in the procedure, these are the calculated values v, P and η, respectively, as indicated in equation (6)

$$\mathbf{t_{8/5}} = \mathbf{aP} + \mathbf{bv} + \mathbf{c\eta} + \mathbf{d} \qquad (6)$$

The independent terms a, b, c, d are the coefficients to calculate for each of the combinations in the design of experiments, constituting the elements of the matrix [β] (Suzuki et al., 1991).

To calculate the matrix of coefficients [β] the matrix equation shown in equation (7) has to be posed.

$$[\boldsymbol{\beta}] = [\mathbf{X^T X}]^{-1}[\mathbf{X^T}][\mathbf{t_{8/5}}] (7)$$

The matrix $[t_{8/5}]$ contains the cooling time values calculated by the thermal simulation model (Estrems et al., 2008) which is based on the Cranck-Nicholson method for solving the differential equation of the heat (Crank, 1947).To calculate each of these cooling times the thermal simulation program is run and as a result the heat map is obtained for each of the welding conditions that have been established. Geometrical data of the base metal are shown in Table 4.

L	0.3	M	Length

B	0.075	M	Width
D	0.002	M	Thickness

Table 4. Geometric data

The values of the physical properties of the stainless steel which has been used in the simulation are shown in Table 5.

T_0	20	ºC	Initial Temperature
k	25	J/m/s/K	Fourier conductivity
T_f	1400	ºC	Temperature of fusion
C_p	630	J/kgK	Calorie Capacity
ρ_0	7500	kg/m^3	Density
ΔH	272000	J/kg	Enthalpies of phase change

Table 5. Physical Properties

The size of the mesh of each of the nodes is indicated in Table 6.

Δx	0.0015	m	Direction of movement
Δy	0.0015	m	Direction perpendicular to the movement

Table 6. Node size

The measurement of $t_{8/5}$ has been calculated at a distance of 1.5 mm from the weld cord axis. In each of the simulations the data in Tables 4, 5 and 6 have been constant, while values of *I, V, v* and *η* are those presented in the matrix of experiments. An example is shown in Figure 1, simulation number 12, and Table 7 shows the value of $t_{8/5}$ for this simulation.

time 800	34.56 s
time 500	59.41s
$t_{8/5}$	**24.84 s**

Table 7. Value of $t_{8/5}$ in experiment 12

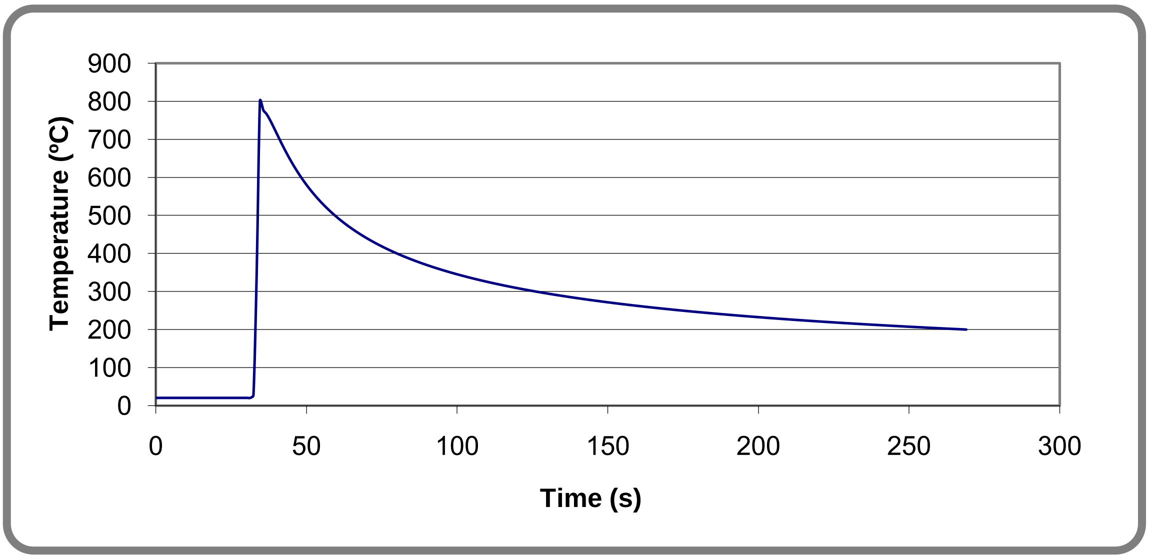

Figure 1. Temperature distribution obtained by numerical modeling in simulation number 12

Similarly, the same is done for each of the experiments, and the matrix of cooling time value is obtained with the mathematical model, as shown in Table 8.

Simulation	Cooling time
Simulation 1	265.684 s
Simulation 2	86.41 s
Simulation 3	162.63 s
Simulation 4	108 s
Simulation 5	54 s
Simulation 6	48.96 s
Simulation 7	131.78 s
Simulation 8	93.61 s
Simulation 9	16.202 s
Simulation 10	86.409 s
Simulation 11	272.164 s
Simulation 12	24.84 s
Simulation 13	95.055 s
Simulation 14	86.409 s
Simulation 15	11.52 s

Table 8.Results $t_{8/5}$ by means of numerical simulation

Finally, the matrix of coefficients [β] obtained is that shown in Table 9.

a	0.08
b	-51852.3
c	209.04
d	-1.06

Table 9. Matrix of coefficients

Thus, the equation $t_{8/5}$ is modelled as established by the equation (8).

$$\mathbf{t_{8/5} = -1.06 + 0.08*Power - 51{,}852.30*Speed + 209.04*Efficiency} \quad \mathbf{(8)}$$

3. Discussion

An analysis of variance is carried out to determine whether the mean differences are statistically significant or not. An analytical method is studied to compare the means of several simulations, based on the construction and analysis of the table called "ANOVA" (Analysis of Variance) (Prat et al., 1994).

With the application of the software "Generated using Sagata Ltd. Software" in the specific case of the first order, the following results are obtained (Montgomery, 2000).

Regression Analysis	*Coefficient*	*Standard Error*	*T statistic*	*p-value*
constant	-1.06	63.60	-0.02	0.987
Power	0.08	0.02	5.00	0.000
Speed	-51852.28	12722.33	-4.08	0.002
Efficiency	209.04	66.24	3.16	0.009

Table 10. Regression analysis

Analysis of Variance	*Degrees of freedom*	*Sum of squares*	*Mean square*	*F-quotient*	*P-value*
Model	3	72367.25	24122.42	17.18	0.000184
Residue	11	15445.77	1404.16		
Total	14	87813.02			

Table 11. Analysis of Variance

Analysing Tables 10 and 11, the output value; the calculated value of statistic F; and its confidence interval; are indicated, the latter being the probability of rejecting the null hypothesis being true. The confidence interval will allow to accept or to reject the null hypothesis, independence between the variables, *P, v, and η,* without having to compare the value of F with an actual value from the statistical tables of a Snedecor's F. The reference value which serves to accept or reject the null hypothesis is the confidence interval. If the confidence interval is greater than 0.05, we will accept the null hypothesis of independence between variables, therefore differential effects will not exist between the welding parameters analyzed. But if the confidence interval is less than 0.05 the null hypothesis will be rejected and the alternative hypothesis accepted, that is to say, we will conclude that there is a dependency relationship of the variables and in this case it can be stated that the different levels of the factor do influence the quantitative variable values.

In the specific case studied, given that the confidence interval **p-value = 0.000184** and this value is less than **α = 0.05** the null hypothesis is rejected and it is accepted that there are differences between the mean values of the variables of the welding parameters analyzed.

From the equation (8) and values of each factor in each of its levels, a value of $t_{8/5}$ is calculated for each of the experiments, as shown in Table 12, where the difference that exists between the values of $t_{8/5}$ calculated by simulation and corresponding to the RSM is established in the form of an error.

Experiment	**Power (W)**	**Speed (m/ s)**	**Efficiency**	**simulated t 8/5**	**RSM t 8/5**	**Error**	**Error (%)**
1	1620	0.0020833	0.8	265.684	187.75	77.94	29.34
2	**1620**	**0.00312465**	**0.6**	**86.41**	**91.944**	**5.53**	**6.4**
3	2530	0.00312465	0.8	162.73	206.55	43.82	26.93
4	**1620**	**0.0020833**	**0.4**	**108**	**104.13**	**3.87**	**3.58**
5	910	0.0020833	0.6	54	89.14	35.14	65.07
6	910	0.00312465	0.8	**48.96**	76.952	27.99	57.17
7	2530	0.004166	0.6	131.78	110.75	21.03	15.96

8	2530	0.00312465	0.4	93.61	122.94	29.33	31.33
9	910	0.004166	0.6	**16.202**	18.853	2.65	16.36
10	1620	0.00312465	0.6	86.409	91.944	5.53	6.4
11	2530	0.0020833	0.6	272.164	218.74	53.42	19.63
12	1620	0.004166	0.4	24.84	3.8607	20.98	84.46
13	**1620**	**0.004166**	**0.8**	**95.055**	**79.755**	**15.3**	**16.1**
14	1620	0.00312465	0.6	86.409	91.944	5.53	6.4
15	910	0.00312465	0.4	**11.52**	6.6643	4.86	42.19

Table 12. Analysis of the error made, adjusted by RSM.

According to the above, the best fit of the response surface is obtained in terms of absolute value of the error for experiment number 9, corresponding to a power of 910 W, a speed of 0.0042 m/s and an efficiency of 0.6. Taking into account the relative error, expressed as a percentage from the simulated value, obtained in experiments 2, 4 and 13, as illustrated in Table 12, it is seen that these three conditions all correspond to the maximum welding power.

Figures 2 to 4 show the response surface of the cooling time, $t_{8/5}$, depending on the factors involved, selected two by two.

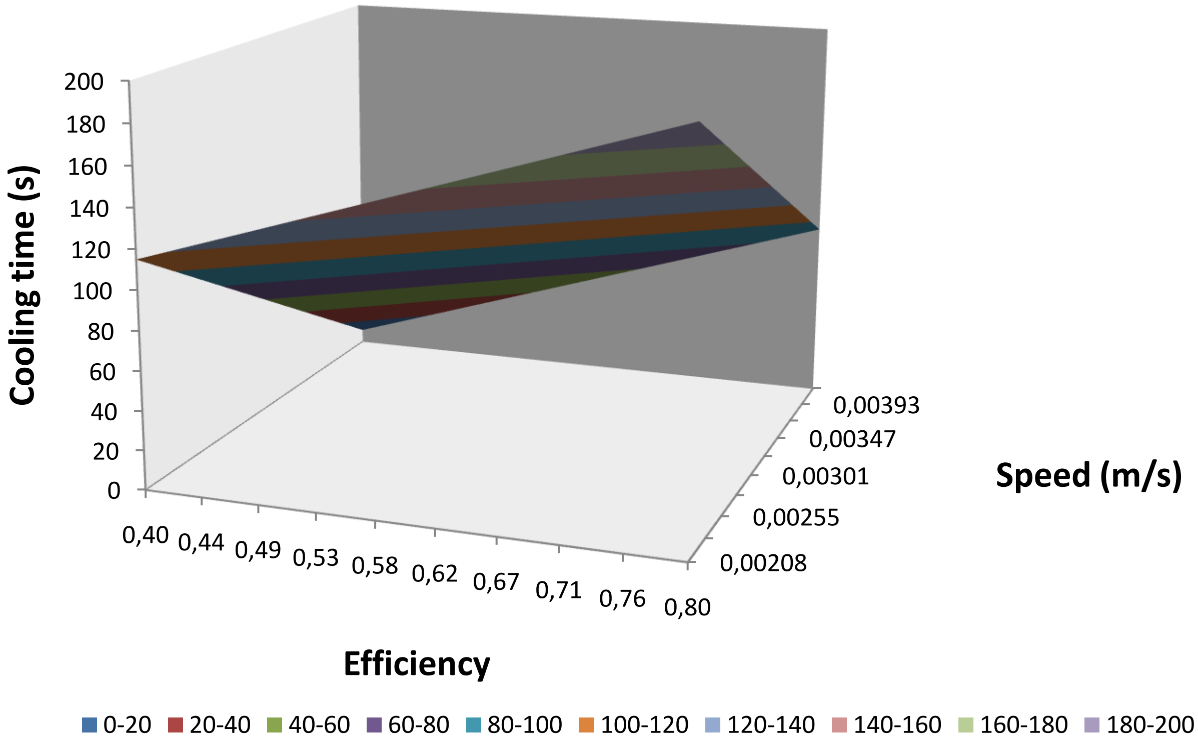

Figure 2. $t_{8/5}$ depending on the thermal performance and the welding speed.

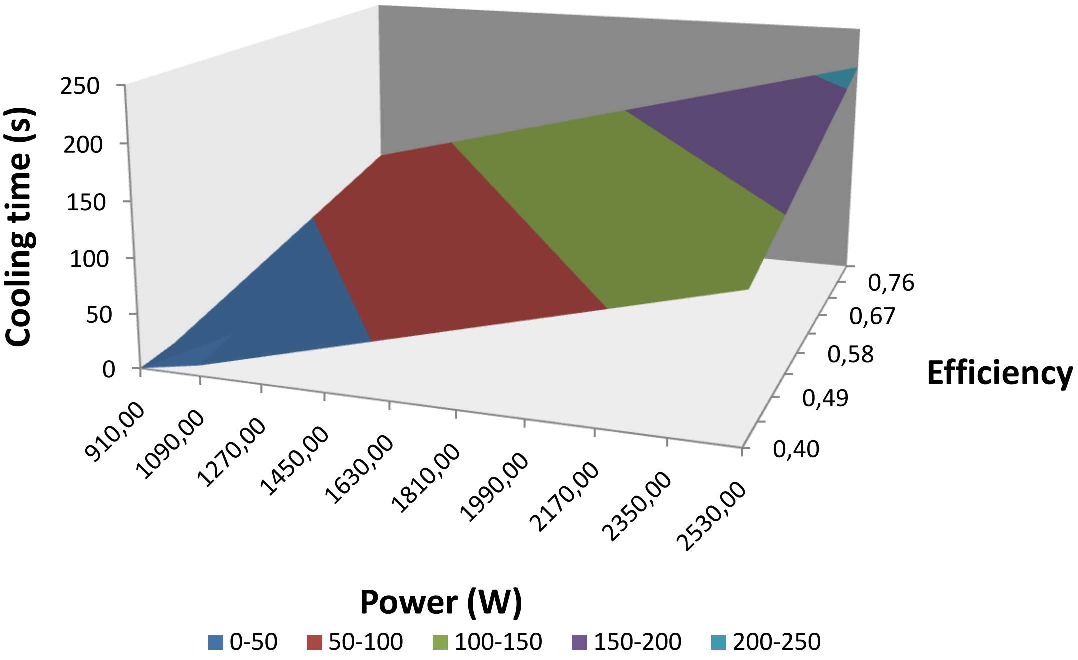

Figure3. t $_{8/5}$ depending on the power and the thermal performance of the welding process.

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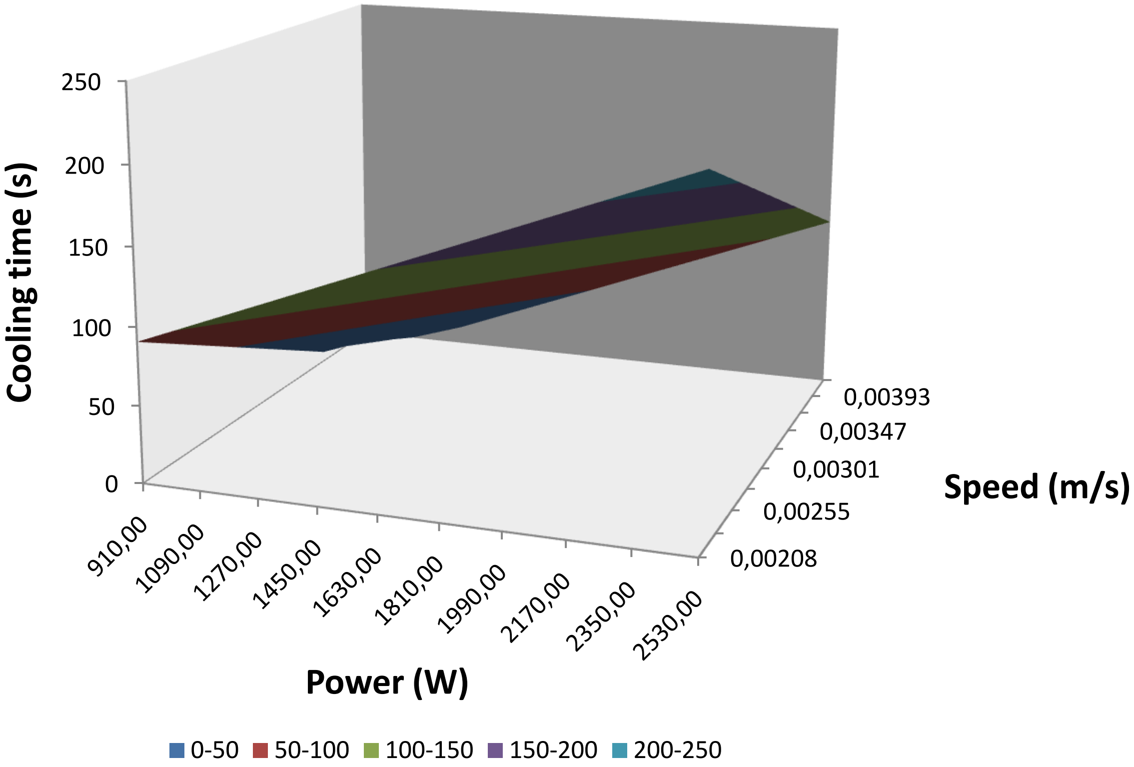

Figure 4. t $_{8/5}$ depending on the power and the welding speed.

Figure 5 shows a relationship between the response of the method, cooling time and the factors analyzed separately, *P, v and η.* As is usual, it is observed that as the power of the electric arc welding machine increases so the cooling time after welding also increases. It is also noted that if the welding speed increases the cooling time decreases, and finally, if the thermal efficiency of the process increases then the cooling time increases.

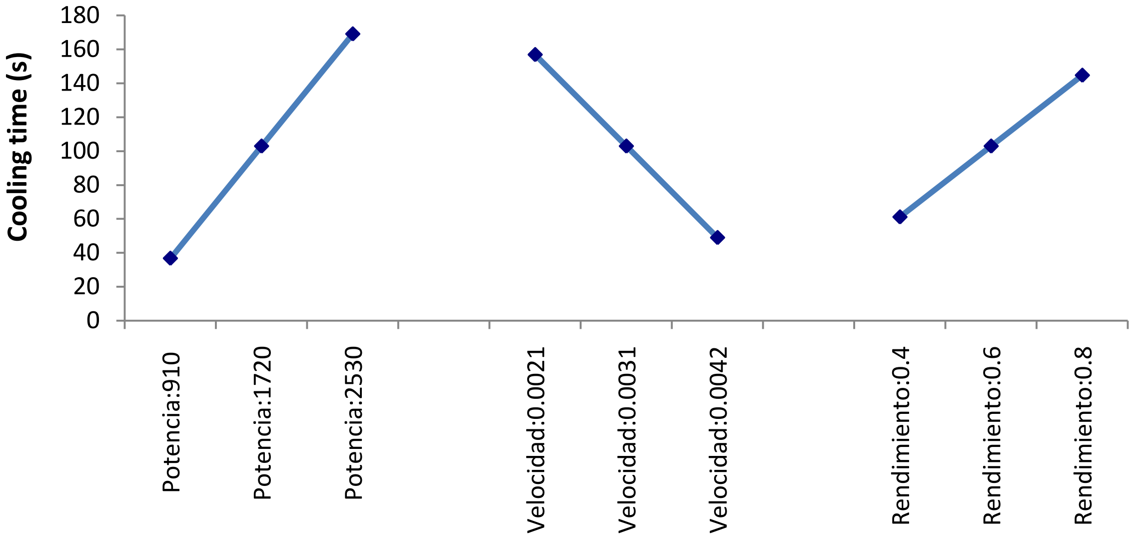

Figure 5. Relation cooling time- factors

From the above, it could be established that for the different efficiencies, the highest values of speed tested, together with those with the lowest power, this would lead to smaller values of $t_{8/5.}$ As can be seen in Table 12, the lowest value for the cooling time is obtained, in every case, for the least power applied. But this does not occur with the welding speed, which adopts a different optimal speed depending on the efficiency that is being considered.

The error values obtained are high in many cases. This suggests a process of refinement in the RSM method consisting in designing new levels of value for the variables in the environment of the optimized value for $t_{8/5,}$ that is to say, for a value of environment of the variables *P, v and η* which make the cooling time minimum, according to equation (8).

4. CONCLUSIONS

In the present work we have applied the methodology of response surface modeling for the different variables involved in the cooling time of the processes of electric arc welding. Specifically, we have obtained an expression that predicts the behavior of the cooling time after welding between 800ºC and 500ºC, $t_{8/5,}$ depending on the variables of arc power; welding speed; and thermal efficiency performance of the operation. The ANOVA analysis has made it possible to establish that these variables significantly affect the cooling time. The fit has been made from values of $t_{8/5}$ obtained by a numerical modeling procedure applied to the welding of austenitic stainless steel sheet and a thickness of less than 3 mm by TIG welding.

The analysis of the error obtained by contrasting the values obtained using the numerical method and with the response surface methodology, RSM, for different values of the variables considered, suggests carrying out a refinement process in the environment of values for the variables that lead to minimum value of $t_{8/5}$ by RSM.

Other process variables such as the geometry of the joint and the existence of preheating, could be considered in the modeling of cooling time.

The application to other materials and arc welding techniques must be performed based on values obtained by numerical methods adjusted accordingly with the specific characteristics of the process and the material, or otherwise from experimental data.

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